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International Policy Coordination in Interdependent Monetary Economies

by
Frederick van der Ploeg

Reprinted from Journal of International Economics,
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Reprint Series
no. 2



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INTERNATIONAL POLICY COORDINATION IN INTERDEPENDENT MONETARY ECONOMIES

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An optimising equilibrium model of two interdependent monetary economies with no other assets except cash, perfect foresight, flexible exchange rates and imperfect substitution between home and foreign goods is analysed. Both non-cooperative and cooperative market-oriented outcomes are time inconsistent, since each government has an incentive to renege and levy a surprise inflation tax. International policy coordination without pre-commitment can be counterproductive even though there are no tax distortions and the provision of public goods is optimal, since it exacerbates the credibility problems perceived by the private sectors and therefore leads to a too low level of real money balances and excessive inflation.

1. Introduction

Recently a great deal of attention has been devoted to international coordination of monetary policies. The pioneering papers on international conflict in monetary stabilisation are Cooper (1969) and Hamada (1976). The latter uses an n -country model with fixed exchange rates, based on the monetary approach to the balance of payments, to compare non-cooperative and cooperative outcomes when each country chooses its own rate of credit expansion to maximise a welfare function that depends on the common rate of inflation and the balance of payments. If the creation of international reserves exceeds the weighted average of desired levels of the balance of payments, monetary expansion is a public bad and competitive outcomes lead to excessive inflation. Hamada (1979) extends the analysis to flexible exchange rates. More recently numerical simulation and differential game theory has been used to discuss the benefits of international policy coordination in analytical two-country models of real-exchange-rate overshooting with sluggish goods and efficient financial markets, perfect capital mobility and flexible exchange rates when preferences depend on output and inflation [e.g. Currie and Levine (1985), Miller and Salmon (1985), Oudiz and Sachs (1985), Başar, Turnovsky and d'Orey (1985)]. Others have done the same with models that incorporate the staggered wage-setting hypothesis [Taylor

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(1985), Carlozzi and Taylor (1985)]. Typically, one finds that non-cooperative monetary policies are too tight over the adjustment period and therefore disinflation occurs excessively fast. However, if Central Banks cannot pre-commit themselves, credibility problems are exacerbated and international coordination may become counterproductive, as established in the pioneering paper of Rogoff (1985). Subsequently, Miller and Salmon (1985) have found an example of counterproductive international policy coordination in numerical simulations of an ad hoc two-country real-exchange-rate overshooting model.

The international coordination games cited above suffer from two problems. The first is the ad hoc specification of macroeconomic multi-country models and welfare functions. The discussion of such games will benefit from using macroeconomic models with microeconomic foundations, because then the welfare functions can be related to well-defined concepts such as consumers' surplus and the derivation of subgame-perfect (or credible) equilibria between each government and its private sector becomes more transparent [e.g. see van der Ploeg (1986b) for an analysis of coordination of optimal taxation in a static two-country model with micro foundations]. This might therefore shed new light on the derivation of time-consistent policies, which is crucial in any discussion of the benefits of international policy coordination. The second problem with previous studies is that they do not allow for a genuine long-run trade-off between inflation and activity. This means that international policy coordination is a transient issue and is concerned with the efficient adjustment towards given long-run targets. However, when there is a long-run conflict in international objectives, there is a more systematic role for coordination of monetary policies. The objectives of this paper are therefore to investigate the desirability of international policy coordination with and without pre-commitment within the context of a classical model of two interdependent monetary economies with micro foundations and a genuine long-run trade-off between inflation and output. In this sense, this paper attempts to reproduce the results derived originally in Rogoff (1985) within a more satisfactory framework. In fact, Rogoff (1985, section 3) informally suggests that unanticipated monetary policy may have beneficial effects in the presence of distortionary taxation of income and this paper attempts to formalise these effects within a two-country world. Kehoe (1986, 1987) has also developed a number of internally consistent models of international policy coordination. They are based on a two-country version, with labour immobility, savings mobility and linear technologies, of Fischer's (1980) model of the time inconsistency of dynamic optimal taxation of capital and labour. Kehoe also obtains the result that coordination without pre-commitment can be counterproductive, but his model is perhaps less realistic than Rogoff's (1985) model, as it abstracts from monetary factors.

Section 2 sets up the micro foundations of a model of two interdependent monetary economies with flexible exchange rates and imperfect substitution between home and foreign goods. It corresponds to a two-country version of the model analysed in Calvo (1978). Section 3 discusses the first-best optimum for the world economy, which gives rise to no distortionary taxes, optimal provision of public goods and Friedman's optimal quantity of money. Section 4 derives the perfect-foresight solution for real money balances and the indirect utilities in terms of tax rates and level of government spending at home and abroad. Section 5 shows that both the cooperative and non-cooperative market-oriented outcomes with pre-commitment are time inconsistent, since each government has an incentive to levy a surprise inflation tax, which permits a decrease in taxes or increase in government spending and thereby increases welfare. This time inconsistency arises despite the fact that each government maximises the utility of the representative household. Section 6 shows that international policy coordination without pre-commitment can be counterproductive, since it exacerbates the credibility constraints vis-à-vis the private sectors and therefore leads to excessive inflation. The point is that competitive decision-making has a built-in disincentive to renege, since a surprise inflation tax induces a depreciation of the real exchange rate and imposes inflation costs. Credible policy coordination does not induce a depreciation and therefore does not have such a disincentive to renege. Section 7 provides a numerical example and section 8 concludes the paper. The appendix criticises the loss of leadership derivation of credible strategies and suggests subgame-perfect strategies as an alternative.

2. A two-country general equilibrium model

This section considers a two-country general equilibrium model with flexible prices and imperfect substitution between home and foreign goods. Households and firms in each country only hold domestic cash and no other domestic or foreign assets. There is, therefore, neither capital nor labour mobility. The government levies distortionary taxes on income from production and also imposes 'inflation taxes' in order to finance the provision of public goods. For simplicity, Cobb-Douglas utility functions and linear technologies are adopted, although the general expressions are also given.

The home household decides on its consumption of home and foreign goods, c_D and c_M , labour supply, L , and holdings of real money balances, m . Its utility function is given by

$$\int_0^T \exp(-\delta t) U(c_D, c_M, L, g, m) dt = \int_0^T \exp(-\delta t) [x_1 \log(c_D) + x_2 \log(c_M)]$$

$$+ \alpha_3 \log(1 - l^s) + \alpha_4 \log(g) + v(m)] dt,$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1, \alpha_i \geq 0, v''(m) > 0, \quad (1)$$

where δ denotes the pure rate of time preference and g denotes the level of public spending. $U(\cdot)$ is assumed to be separable in c_D , c_M , l^s , g and m . Feenstra (1986) argues that having real money balances in the utility function can be consistent with a transaction technology. There exists an m , say m_f , such that $U_m = v'(m) = 0$ and this will be called Friedman's optimal quantity of money (OQM) [Friedman (1969)]. The home household maximises (1) subject to its budget constraint:

$$\dot{m} = (1 - \tau)(wl^s + z) - c_D - ec_M - \pi m, \quad (2)$$

where τ , w , z , e and π denote the tax rate on labour income, the real wage rate, real profits, the real exchange rate and the rate of home inflation, respectively. It follows that $c_D = C_D(\bar{\lambda}) = \alpha_1/\bar{\lambda}$, $c_M = C_M(e\bar{\lambda}) = \alpha_2/e\bar{\lambda}$, $l^s = L^s((1 - \tau)w\bar{\lambda}) = 1 - \alpha_3/(1 - \tau)w\bar{\lambda}$, so that home goods, foreign goods and leisure are normal as preferences are separable. Also,

$$\dot{\lambda} = (\delta + \pi^e)\bar{\lambda} - v'(m), \quad \lim_{t \rightarrow \infty} \exp(-\delta t) m(t) \dot{\lambda}(t) = 0, \quad (3)$$

where $\dot{\lambda}$ is the marginal value of money balances, must hold. Hence, home consumption of home goods, foreign goods and leisure are a decreasing function of the marginal value of money balances. Also, imports fall when the real exchange rate depreciates and the supply of labour increases when the after-tax real wage increases. Eq. (3) says that the representative household sets its marginal utility of money balances, $v'(m)/\bar{\lambda}$ (expressed in monetary units), to the rate of time preference, δ , minus the expected real instantaneous return on holding cash balances, $-\pi^e$, minus the rate of appreciation of the marginal value of money balances, $-\dot{\lambda}/\bar{\lambda}$.

The home economy has a very rudimentary production sector. There is a representative firm with a non-increasing-returns-to-scale technology, $f(\cdot)$, which chooses its demand for labour, l^d , to maximise profits, z , under perfect competition. Hence, the firm hires labour until the marginal productivity of labour equals the real wage, $f'(l^d) = w$. Labour market equilibrium, $l^d = l^s$, then gives $l = L^s((1 - \tau)f'(l)\bar{\lambda}) \equiv L((1 - \tau)\bar{\lambda})$, $L' > 0$. Money market equilibrium gives $\mu = \pi + \dot{m}/m$, so that the growth in the nominal supply of money, μ , is matched with the growth in the nominal demand for money. The government budget constraint of the home economy can be written as

$$\mu m = d \equiv g - \tau f(l), \quad (4)$$

so that the public sector deficit, the excess of public spending over tax revenues, has to be financed (in the absence of any other form of government assets) by printing money.

The foreign economy has identical tastes and preferences and the same population size. All foreign variables and expressions are denoted by an asterisk. Goods market equilibrium follows from the national accounting identity:

$$f(l) = c_D + g + c_M^* \quad (5)$$

Finally, exchange market equilibrium follows from balanced trade:¹

$$C_M^*(\tilde{\lambda}^*, e) = e C_M(e\tilde{\lambda}), \quad (6)$$

which gives the equilibrium real exchange rate:

$$e = E(\tilde{\lambda}, \tilde{\lambda}^*) = \tilde{\lambda}^* / \tilde{\lambda}, \quad (7)$$

where $E_{\tilde{\lambda}} = e^2 C_M' / c_M (\eta + \eta^* - 1) < 0$ and $E_{\tilde{\lambda}^*} = -C_M^* / c_M^* (\eta + \eta^* - 1) > 0$ as the Marshall-Lerner condition, $-(\tilde{\lambda} e C_M' / c_M) - (\tilde{\lambda}^* C_M^* / e c_M^*) - 1 = \eta + \eta^* - 1 > 0$, where η and η^* are the import and export elasticities, is assumed to be satisfied. Since $E(\cdot)$ is homogeneous of degree zero in $\tilde{\lambda}$ and $\tilde{\lambda}^*$, $c_M = C_M(E(\tilde{\lambda}, \tilde{\lambda}^*)\tilde{\lambda}) \equiv \hat{C}_M(\tilde{\lambda}^*) = \alpha_2 / \tilde{\lambda}^*$. An increase in the marginal value of domestic (foreign) money balances leads to decreased imports (exports) for the home economy and the resulting incipient trade surplus (deficit) is choked off by an appreciation (depreciation) of the real exchange rate.

When the above relationships are combined with the perfect foresight hypothesis, $\pi^* = \pi$, one obtains the perfect foresight equilibrium (PFE) which gives the endogenous variables conditional on expectations of current and future values of the government's policy instruments. In particular, the demand for real money balances depends negatively on the expected inflation rate and therefore the price level is history-independent and jumps in order to maintain money market equilibrium. Since the price level changes instantaneously in response to news about future changes in government policy, real money balances also change instantaneously. The objective of each government is to choose its fiscal and monetary policies to maximise the utility of its representative consumer subject to the constraints of the PFE.

¹Combining the household and government budget constraint and the goods market equilibrium condition, one obtains the condition for balanced trade. Hence, one of (2), (4), (5) and (6) is redundant.

3. First-best optimum for the world economy

Before a discussion of coordinated and uncoordinated optimal taxation of interdependent decentralised market economies takes place, it seems useful to briefly discuss the first-best optimum for the world economy. The first-best optimum is useful as it gives an upper bound on the welfare that can be obtained in a system of interdependent market economies. It can be obtained as follows:

$$\max_{\substack{c_D, c_M, l, g, m \\ c_D^*, c_M^*, l^*, g^*, m^*}} \int_0^x \exp(-\delta t) [U(c_D, c_M, l, g, m) + U^*(c_D^*, c_M^*, l^*, g^*, m^*)] dt. \quad (8)$$

subject to the world balance condition:

$$f(l) + f^*(l^*) = c_D + g + c_M + c_D^* + g^* + c_M^*. \quad (9)$$

Hence, the world planning authority maximises a utilitarian global welfare function subject to ensuring that the world demand for goods matches the world supply of goods. This yields:

$$U_{c_D} = U_{c_M} = \frac{-U_l}{f'(l)} = U_g = U_{c_D^*}^* = U_{c_M^*}^* = \frac{-U_{l^*}^*}{f'^*(l^*)} = U_{g^*}^* = \xi \quad (10)$$

and

$$v'(m) = v'^*(m^*) = 0, \quad (11)$$

where ξ is the marginal utility of an extra unit of output. The marginal rate of substitution between home and foreign consumption of home, public and foreign goods is unity and the marginal rate of substitution between the consumption of goods and leisure is the marginal productivity of labour. This corresponds to no tax distortions in a system of interdependent market economies ($\tau = \tau^* = 0$). Eq. (11) says that the marginal utility of home and foreign money balances is zero, so that Friedman's OQM pertains ($m = m^* = m_F$).

It follows that the (non-market) first-best optimum for the world economy can be characterised by three conditions: (i) the marginal rate of substitution between public and private goods is unity; (ii) no tax distortions; and (iii) Friedman's OQM. Note that the implied monetary growth rate is $\mu = g/m_F$, so that, in general, the full liquidity rule, $\mu = -\delta$, does not hold in a first-best optimum. The full liquidity rule is only relevant when market equilibrium is a constraint on policy; in a market outcome it is feasible to ensure full

liquidity when the government of each country can issue bonds as well as print money [cf. Turnovsky and Brock (1980)].

For the special case of Cobb–Douglas preferences and linear technologies, $f(l) = \beta l$, one obtains $\xi = 1/\beta$, $c_D = c_D^* = \alpha_1 \beta$, $c_M = c_M^* = \alpha_2 \beta$, $g = g^* = \alpha_4 \beta$, $1 - l = 1 - l^* = \alpha_3$ and therefore

$$U = U^* = \alpha_1 \log(\alpha_1) + \alpha_2 \log(\alpha_2) + \alpha_3 \log(\alpha_3 \beta) + \alpha_4 \log(\alpha_4) \\ + \log(\beta) + r(m_t) \equiv U^F. \quad (12)$$

The associated monetary growth rates are given by $\mu \equiv \mu^* = \alpha_4 \beta / m_t$.

4. Derivation of the perfect-foresight equilibrium

Substitution of the consumption and labour supply functions into (5),

$$f(L((1 - \tau)\lambda)) = C_D(\lambda) + g + \hat{C}_M^*(\lambda), \quad (13)$$

yields the marginal utility of money balances:

$$\lambda = A(\tau, g) = \{ \alpha_1 + \alpha_2 + \alpha_3 / (1 - \tau) \} / (\beta - g), \quad (14)$$

where $A_g = \{ f' L'(1 - \tau) - C_D' - \hat{C}_M^{*'} \}^{-1} > 0$ and $A_\tau = f' L' \lambda A_g > 0$. Upon substitution of (14) into the labour supply function one obtains:

$$((1 - \tau)A(\tau, g)) = \hat{L}(\tau, g) = 1 - (\beta - g)\alpha_3 / \{ \beta((\alpha_1 + \alpha_2)(1 - \tau) + \alpha_3) \}, \quad (15)$$

and therefore the public sector deficit, (4), can be written as

$$d = g - \tau f(\hat{L}(\tau, g)) \equiv D(\tau, g) \\ = \{ g(\alpha_1 + \alpha_2 + \alpha_3) - \tau \beta (\alpha_1 + \alpha_2) \} / \{ \alpha_1 + \alpha_2 + \alpha_3 / (1 - \tau) \}. \quad (16)$$

Similarly, the consumption functions can be written as

$$c_D = C_D(A(\tau, g)) \equiv \hat{C}_D(\tau, g) = \alpha_1 (\beta - g) / \{ \alpha_1 + \alpha_2 + \alpha_3 / (1 - \tau) \} \quad (17)$$

and

$$c_M = \hat{C}_M(A^*(\tau^*, g^*)) \equiv \hat{C}_M^*(\tau^*, g^*) = \alpha_2 (\beta - g^*) / \{ \alpha_1 + \alpha_2 + \alpha_3 / (1 - \tau^*) \}. \quad (18)$$

Upon substitution of (15), (17) and (18) into (1), one can write the indirect instantaneous utility function as

$$\begin{aligned}
 U(\hat{C}_D(\tau, g), \hat{C}_M(\tau^*, g^*), \hat{L}(\tau, g), g, m) &\equiv V(\tau, g, \tau^*, g^*) + v(m) \\
 &= \alpha_0 - (\alpha_1 + \alpha_3) \log \{ \alpha_1 + \alpha_2 + \alpha_3 / (1 - \tau) \} - \alpha_3 \log (1 - \tau) \\
 &\quad - \alpha_2 \log \{ \alpha_1 + \alpha_2 + \alpha_3 / (1 - \tau^*) \} + (\alpha_1 + \alpha_3) \log (\beta - g) + \alpha_4 \log (g) \\
 &\quad + \alpha_2 \log (\beta - g^*) + v(m),
 \end{aligned} \tag{19}$$

where $\alpha_0 \equiv \alpha_1 \log(\alpha_1) + \alpha_2 \log(\alpha_2) + \alpha_3 \log(\alpha_3/\beta)$. Substitution of (14), (16) and $\pi^e = \pi = \mu - \dot{m}/m$ into (3) yields:

$$\begin{aligned}
 \dot{m} &= m \{ \delta + D(\tau, g)/m - (v'(m) + A_\tau \dot{\tau} + A_g \dot{g})/A(\tau, g) \} \\
 &\equiv \psi(m, \tau, \tau^*, g, g^*), \quad m(0) = \text{free}.
 \end{aligned} \tag{20}$$

Eq. (20) can be inverted to give the demand for real money balances as a decreasing function of the tax rate, level of public spending and the expected inflation rate, so that it gives a micro foundation of a Cagan-type money-demand schedule. It is assumed that $\psi_m = -(d + m^2 \tau''/\lambda)/m > 0$, which will be the case as long as the public sector deficit or the monetary growth rate is not too large. Eqs (19) and (20) describe the reduced form of the perfect-foresight equilibrium (PFE). The instability of (20), i.e. $\psi_m > 0$, reflects the saddlepoint property of the PFE, for in the PFE the price level and thus the stock of real money balances are forward-looking variables and depend upon expectations of current and future values of the government's policy instruments.

An increase in the home tax rate reduces the opportunity cost of leisure and thus reduces the supply of labour and goods. This results in an increase in the marginal utility of home money balances, which increases the home supply of labour and output and dampens home consumption of home and foreign goods. The resulting surplus in the current account leads to an appreciation of the real exchange rate, which dampens foreign consumption of home goods. The above process continues until equilibrium in the goods and foreign exchange markets is attained. The public sector deficit improves, despite the decrease in the tax base, so that the monetary growth rate can be reduced. The net effect on the utility of the representative household is ambiguous, since the fall in consumption worsens utility and the increase in leisure improves utility.

An increase in home public spending also leads to an excess demand for

home goods, which is choked off by an increase in home supply and a fall in home and foreign consumption of home goods (induced by an increase in the marginal utility of money balances). The net effect on utility is ambiguous, because the falls in consumption and leisure deteriorate utility and the increased provision of public goods increases utility. The public sector deficit increases, despite the increase in the tax base.

Now consider the externalities induced by changes in foreign policy. An increase in the foreign tax rate or level of public spending leads to a reduction in the foreign demand for home goods. The incipient trade deficit is choked off by a depreciation of the home market exchange rate, which reduces home consumption of foreign goods and therefore worsens home utility. Note that these externalities only play a role in the indirect utility function and do not affect the dynamics of the PFE, (20).

5. Optimal taxation with pre-commitment of interdependent monetary economies

5.1. *Non-cooperative policies with pre-commitment*

The problem of each government is to choose the tax rate and level of public spending in such a way as to maximise the discounted utility of the representative household (ignoring distributional considerations) subject to the PFE:

$$\max_{\tau, g} \int_0^{\infty} \exp(-\delta t) [V(\tau, g, \tau^*, g^*) + v(m)] dt \quad (21)$$

subject to (20). The associated monetary growth rates follow from the government budget constraint, i.e. $\mu = D(\tau, g)/m$. In this section it is assumed that each government can pre-commit itself to the announced strategies. This corresponds to committing itself, via constitutional law or institutional constraints, to a particular path of tax rates and levels of public spending. For deterministic economies, this implies a pre-commitment to a path of monetary growth rates. The non-cooperative or competitive policies are discussed first.

The competitive policies assume that each government takes the policies adopted by the other government as given, so that the resulting equilibrium between the two governments is an open-loop Nash equilibrium.² The separability of the indirect utility function, (19), means that the Nash equilibrium solution coincides with isolationist policies, i.e. the policies each

²The game between each government and its constituents is modelled as a Stackelberg game with pre-commitment, which yields an open-loop Stackelberg equilibrium solution [Simaan and Cruz (1973a)].

government would pursue when ignoring the repercussions of foreign policies. The optimality conditions follow from Pontryagin's Maximum Principle: (20),

$$H_m = v'(m) + \dot{\lambda}_1 \psi_m = \delta \dot{\lambda}_1 - \dot{\lambda}_1, \quad \dot{\lambda}_1(0) = 0, \quad (22)$$

$$H_\tau = V_\tau + \dot{\lambda}_1 \psi_\tau = \delta \dot{\lambda}_2 - \dot{\lambda}_2, \quad \dot{\lambda}_2(0) = 0, \quad (23)$$

$$H_g = V_g + \dot{\lambda}_1 \psi_g = \delta \dot{\lambda}_3 - \dot{\lambda}_3, \quad \dot{\lambda}_3(0) = 0, \quad (24)$$

$$H_\tau = \dot{\lambda}_1 \psi_\tau + \dot{\lambda}_2 = 0, \quad (25)$$

and

$$H_g = \dot{\lambda}_1 \psi_g + \dot{\lambda}_3 = 0, \quad (26)$$

where the (undiscounted) Hamiltonian is defined as $H(\cdot) \equiv V(\cdot) + v(m) + \dot{\lambda}_1 \psi(\cdot) + \dot{\lambda}_2 \tau + \dot{\lambda}_3 g$. The stock of real money balances is not constrained by its past history and can take on any value at the beginning of the planning horizon. It follows that the marginal contribution to economic welfare of the initial stock of real money balances must be zero, hence $\dot{\lambda}_1(0) = 0$ and, from (25)–(26), $\dot{\lambda}_2(0) = \dot{\lambda}_3(0) = 0$ [cf. Calvo (1978)]. If the government were to re-optimize at a later date, say $\bar{t} > 0$, then the marginal utility of money balances would be set to zero at that date, $\dot{\lambda}_1(\bar{t}) = 0$. It follows that the optimal policies of the government are time consistent [see Kydland and Prescott (1977)] only if $\dot{\lambda}_1(t) = 0, \forall t \geq 0$. Since then there would never be an incentive for the government to renege on its announced strategies.

Proposition 1. The optimal non-cooperative policies are time inconsistent.

Proof. If $\dot{\lambda}_1(t) = 0, \forall t$, then $\dot{\lambda}_2(t) = \dot{\lambda}_3(t) = 0, \forall t$, and from (22)–(24) $v'(m) = V_\tau = V_g = 0, \forall t$. It follows that $m(t) = m_F, \forall t, \tau(t) = \alpha_2/(\alpha_1 + \alpha_2) \equiv \tau^N, \forall t$, and $g(t) = \alpha_4\beta/(1 - \alpha_2) \equiv g^N, \forall t$. Substitution into (20) yields:

$$\dot{m} = \psi(m_F, \tau^N, 0, g^N, 0) = \delta m_F + (\alpha_4 - \alpha_2)\alpha_1\beta/(1 - \alpha_2)(\alpha_1 + \alpha_2) \neq 0,$$

so that the non-cooperative plan cannot be time consistent. \square

The reason for this time inconsistency is that, in the absence of binding contracts, pre-commitment or reputational forces, the government has an incentive to increase the monetary growth rate and inflation in order to erode the real value of money balances and thereby increase economic welfare. The reason behind this incentive to renege is that the increase in seignorage revenues permits an instantaneous decrease in taxes and increase in government spending, which improves economic welfare. Note that this

time inconsistency arises despite the fact that there is no conflict between private and public objectives, since each government maximises the utility of its representative household.³ The problem with the government reneging on its announced strategies is that expectations of the private sector are not fulfilled and therefore the government loses reputation. Unless there are binding contracts that tie the hands of the governments, the non-cooperative policies found from (20) and (22)–(26) will not be credible.

5.2. *International policy coordination*

In the previous section each government decided on its own policies independent of the policies adopted by the government of the other country, which resulted in an open-loop Nash equilibrium with pre-commitment for the world economy. Coordinated policies may be found from maximising a utilitarian welfare function for the world economy:

$$\max_{\tau, g, \tau^*, g^*} \int_0^T \exp(-\delta t) [V(\tau, g, \tau^*, g^*) + v(m) + V^*(\tau^*, g^*, \tau, g) + v(m^*)] dt \quad (27)$$

subject to the PFE of the home and foreign economy. The efficient policies follow from (20), (22),

$$V_\tau + V_\tau^* + \lambda_1 \psi_\tau = \delta \lambda_2 - \dot{\lambda}_2, \quad \lambda_2(0) = 0, \quad (28)$$

$$V_g + V_g^* + \lambda_1 \psi_g = \delta \lambda_3 - \dot{\lambda}_3, \quad \lambda_3(0) = 0, \quad (29)$$

(25) and (26), and similar relationships for the foreign economy. The main difference with non-cooperative strategies is that the world planner takes account of the negative effects of higher taxes and public spending on foreign welfare. The result of these externalities is that taxes and public spending are higher with the non-cooperative outcomes than with the coordinated outcomes. It can also easily be shown that the coordinated policies announced by the world planner are not credible, unless they are backed up by binding contracts to prevent the world planner from reneging on the home and foreign private sector agents and by binding contracts to prevent each country from deviating from the cooperative agreements.

Proposition 2. The optimal policies under international policy coordination with pre-commitment are time inconsistent.

Proof. If $\dot{\lambda}_1(t) = 0, \forall t$, then $\dot{\lambda}_2(t) = \dot{\lambda}_3(t) = v'(m) = 0, \forall t$, and from (28)–(29)

³The problem of a surprise inflation levy would be more severe if there were non-indexed bonds in the model.

$V_t + V_t^* = V_g + V_g^* = 0$. It follows that $m(t) = m_F$, $\forall t$, $\tau(t) = 0 < \tau^N$, $\forall t$, and $g(t) = \alpha_4 \beta < g^N$, $\forall t$. Substitution into (20) yields:

$$\dot{m} = \psi(m_F, 0, 0, \alpha_4 \beta, 0) = \delta m_F + \alpha_4 \beta > 0,$$

which cannot be consistent with $m(t) = m_F$, $\forall t$. Hence, the optimal coordinated policies must be time inconsistent. \square

6. Time consistency, loss of leadership and optimal taxation

6.1. Non-cooperative policies without pre-commitment

One way to obtain a time consistent plan for optimal taxation is for each government to treat its price level or, alternatively, its stock of real money balances as a predetermined rather than as a jump variable. This corresponds to an open-loop Nash equilibrium between the government and the private sector and therefore this solution to the time inconsistency problem has been referred to as the 'loss of leadership' solution [Buiter (1983), Miller and Salmon (1985)]. The government effectively resigns itself to the fact that it will be unable to pre-commit itself and therefore unable to manipulate the price level or the holdings of real money balances. This means that the shadow price of real money balances, λ_1 , is always set to zero, so that (22) is replaced by $\lambda_1(t) = 0$, $\forall t \geq 0$.

The loss of leadership solution or the open-loop Nash equilibrium without pre-commitment leads to $\tau(t) = \tau^N$ and $g(t) = g^N$ (see Proposition 1). It follows from (17), (18) and (15) that consumption of home goods, imports and employment are given by

$$c_D = \beta \alpha_1^2 / (\alpha_1 + \alpha_2)(1 - \alpha_2) \equiv c_D^N < \beta \alpha_1, \quad (30)$$

$$c_M = \beta \alpha_1 \alpha_2 / (\alpha_1 + \alpha_2)(1 - \alpha_2) \equiv c_M^N < \beta \alpha_2, \quad (31)$$

where

$$l = (\alpha_1 + \alpha_4) / (1 - \alpha_2) \equiv l^N < 1 - \alpha_3, \quad (32)$$

respectively. The distortions caused by direct taxes reduce the opportunity cost of leisure, so that labour supply is less than in the first-best optimum (or than in the pre-commitment outcome). Hence, output and consumption of home and foreign goods are below the levels attained in the first-best optimum. The provision of public goods is excessive relative to the amount provided in the first-best optimum ($g^N > \alpha_4 \beta$). The public sector deficit (16) is given by

$$d = \beta(\alpha_4 - \alpha_2) \alpha_1 / (\alpha_1 + \alpha_2)(1 - \alpha_2) \equiv d^N. \quad (33)$$

Note that there is a public sector surplus when the weight attached to imported goods, α_2 , exceeds the weight attached to public goods, α_4 . The dynamic behaviour of real money balances follows from (20), that is

$$\dot{m} = \delta m + d^N - \beta \alpha_1 m v'(m) / (\alpha_1 + \alpha_2)(1 - \alpha_2), \quad m(0) = \text{free}. \quad (34)$$

For a specific functional form of $v(m)$, say⁴

$$v(m) = \alpha_5 [m_F \log(m) - m], \quad (35)$$

one obtains the equilibrium stock of real money balances:

$$\lim_{t \rightarrow \infty} m(t) = \beta (\alpha_2 - \alpha_4 + \alpha_5 m_F) \alpha_1 / [(\alpha_1 + \alpha_2)(1 - \alpha_2)\delta + \beta \alpha_1 \alpha_5] \equiv m^N. \quad (36)$$

If the public sector's accounts are approximately balanced or in deficit [$\alpha_2 - \alpha_4 < (\alpha_1 + \alpha_2)(1 - \alpha_2)\delta / (\alpha_1 \beta) \approx 0$], the stock of real money balances is less than the optimal quantity of money. In fact, it will also be less than the stock of real money balances that obtains when both governments can pre-commit themselves. The reason is that the loss of leadership solution recognises that the non-cooperative policies of the two governments must be consistent, so that there must be no incentive for any government to renege by imposing a surprise inflation tax once money balances have been accumulated. In the loss of leadership solution, private sector agents accumulate less money balances, so that neither government has an incentive to renege. Clearly, the resulting non-cooperative outcome yields lower welfare than if pre-commitment was feasible.

6.2. Counterproductive international policy coordination

Now consider the loss of leadership solution when there is international coordination of macroeconomic policies and absence of pre-commitment. This means that the world planning authority takes the home and foreign levels of real liquidity as given at the time of the determination of the optimal policies. Hence, $\dot{\lambda}_1(t) = \dot{\lambda}_2(t) = \dot{\lambda}_3(t) = 0$, $\forall t \geq 0$, implies from (28) and (29) $V_t^* + V_t^{**} = 0$ and $V_g^* + V_g^{**} = 0$. It follows that no distortionary taxes are levied, $\tau = \tau^* = 0$, $\forall t \geq 0$, and that the provision of public goods is below the level provided under non-cooperative policy formulation with pre-commitment, $g = g^* = \alpha_4 \beta < g^N$. In fact, the real levels of consumption of home and foreign goods, public expenditure and leisure are exactly the same as the levels attained in the first-best optimum and therefore exceed the levels

⁴ This form encapsulates the logarithmic utility function, $v(m) \equiv \alpha_5 \log(m)$, where $\alpha_5 \equiv \alpha_5 m_1 > 0$ and $\alpha_5 \rightarrow 0$, which corresponds to an infinite OQM, $m_1 \rightarrow \infty$.

attained under competitive policy formulation with loss of leadership. The dynamics of real money balances under coordination with loss of leadership follow from (20), that is

$$\dot{m} = \delta m + \beta \alpha_4 - \beta m v'(m), \quad m(0) = \text{free}. \quad (37)$$

The steady-state stock of real money balances is given by

$$\lim_{t \rightarrow \infty} m(t) = \beta(\alpha_5 m_F - \alpha_4) / (\delta + \beta \alpha_5) \equiv m^C < m_F, \quad (38)$$

which is less than the OQM and therefore international policy coordination with loss of leadership is obviously inferior to the first-best optimum:

$$U = U^* = U^F + v(m^C) - v(m_F) \equiv U^C < U^F, \quad (39)$$

where U^C denotes asymptotic welfare. Obviously, this is due to steady-state inflation, $\mu = \alpha_4 \beta / m^C \equiv \mu^C$, being higher than in the first-best optimum.

The non-cooperative Nash equilibrium without pre-commitment (see sub-section 6.1) gives rise to an asymptotic welfare of

$$\begin{aligned} U = U^* = U^F - \log(1 - \alpha_2) - (\alpha_1 + \alpha_2) \log((\alpha_1 + \alpha_2)/\alpha_1) + v(m^N) - v(m_F) \\ \equiv U^N < U^F. \end{aligned} \quad (40)$$

The welfare under this non-cooperative outcome with loss of leadership is less than the welfare obtained in the first-best optimum, because the levy of distortionary taxes leads to a smaller labour supply and therefore to less consumption of home and foreign goods and because the holdings of real money balances are unlikely to coincide with Friedman's OQM [unless the parameters satisfy $\alpha_2 - \alpha_4 = (\alpha_1 + \alpha_2)(1 - \alpha_2)\delta/(\alpha_1\beta)$]. There is a welfare loss despite leisure and the provision of public goods being higher than in the first-best optimum.

An unambiguous comparison of welfare under the non-cooperative outcome without pre-commitment and welfare under the cooperative outcome without pre-commitment is, however, not possible. Even though international policy coordination in the absence of pre-commitment eliminates the distortionary effects of taxation and therefore increases welfare, it may well lead to a lower level of real money balances and therefore decrease welfare. If the latter effect dominates, international policy coordination is counterproductive. This is particularly likely to occur for small discount rates, since then

$$m^C/m^N \approx (\alpha_5 m_F - \alpha_4) / (\alpha_5 m_F + \alpha_2 - \alpha_4) < 1, \quad (41)$$

and this may cause coordination to be futile, especially when countries are engaged heavily in international trade (high α_2). Since the public sector deficit is higher under international policy coordination than under competitive decision-making without pre-commitment ($d^N < \alpha_4/\beta$), monetary growth and inflation are likely to be higher under cooperation as well.

The reason that international policy coordination may be counterproductive is that it exacerbates the credibility problems the governments face vis-à-vis the private sectors [cf. Rogoff (1985)]. Competitive decision-making has a built-in check on each government's incentive to renege by imposing a surprise inflation tax in an attempt to erode the real value of money balances. Thus, if there is a unilateral expansion of the monetary growth rate, the marginal value of real money balances (λ) falls, consumption of home and foreign goods increases and the resulting incipient balance of payments deficit is choked off by a depreciation of the real exchange rate. The depreciation leads to inflation costs, which provides a disincentive for governments engaged in non-cooperative decision-making to renege. Under international policy coordination such a built-in check does not exist, since when both governments multilaterally renege and increase their monetary growth rates there will be no depreciation of the real exchange rate and therefore no induced inflation costs. Under international policy coordination the private sectors perceive that the governments have a greater incentive to renege and are therefore forced to have higher monetary growth and inflation rates than under non-cooperative decision-making. This can render international policy coordination futile.

7. A numerical example

Table 1 presents a numerical example that compares a first-best optimum with coordinated and competitive outcomes with and without pre-commitment. The first-best optimum and coordination under loss of leadership both have the OQM and no tax distortions, yet the latter is vastly inferior due to much higher inflation eroding the holdings of real money balances. Decentralised decision-making under loss of leadership yields higher welfare than coordination (a welfare loss of 1.797 rather than 1.885), despite the fact that both governments levy distortionary taxes and therefore cut output, employment and consumption of home and foreign goods. The reason why international policy coordination is counterproductive is that there is no longer a disincentive to renege, since when both governments impose a multilateral surprise inflation tax there will be no induced depreciation of the real exchange rate and therefore no inflation costs. In other words, coordination removes the threat of a depreciation and therefore eliminates a disincentive to pursue an inflationary strategy. It follows that coordination under loss of leadership leads to higher inflation rates and

Table 1

Coordinated and competitive outcomes for interdependent monetary economies with flexible exchange rates.^a

	First-best optimum	Pre-commitment		Loss of leadership	
		Coordinated	Competitive	Coordinated	Competitive
Tax rate, τ		0.197	0.296	0.0	0.364
Public spending, g	0.2	0.191	0.255	0.2	0.250
Public sector deficit, d		0.052	0.045	0.2	0.0
Real money balances, m	1.0	0.944	1.020	0.4	0.761
Monetary growth rate, μ	0.2	0.055	0.044	0.5	0.0
Consumption of home goods, c_D	0.35	0.329	0.288	0.35	0.278
Consumption of foreign goods, c_M	0.2	0.188	0.165	0.2	0.159
Leisure, $1-l$	0.25	0.292	0.292	0.25	0.313
Indirect utility, V	-1.358	-1.362	-1.377	-1.358	-1.383
Utility of m , v	-0.4	-0.4	-0.4	-0.527	-0.414
Total welfare, $U = V + v$	-1.758	-1.762	-1.777	-1.885	-1.797
Welfare ranking	I	II	III	V	IV

Parameters: $\alpha_1 = 0.35$; $\alpha_2 = 0.2$; $\alpha_3 = 0.25$; $\alpha_4 = 0.2$; $\alpha_5 = 0.4$; $m_1 = 1.0$; $\beta = 1.0$; $\phi = 0.1$.

^aThe utility of real money balances, $v(m)$, and total welfare of the representative household, $U(c_D, c_M, 1-l, g, m)$, are evaluated at the steady-state outcomes of m .

lower holdings of real money balances than decentralisation, which makes coordination futile. Note that competitive decision-making without pre-commitment leads to a balanced public sector deficit in this example, because $z_2 = x_4 (= 0.2)$. Both coordination and decentralisation with pre-commitment are superior to decentralisation without pre-commitment and a fortiori coordination without pre-commitment, because they lead to less tax distortions and higher stocks of real money balances. Also, coordination with pre-commitment leads to smaller tax rates, higher monetary growth rates and higher welfare than decentralisation with pre-commitment.⁵

8. Concluding remarks

A two-country general equilibrium model with imperfect substitution between home and foreign goods, agents that have home cash as their only asset and flexible exchange rates has been formulated. The model has micro foundations, so that the analysis of welfare can be conducted in terms of consumers' surplus. Also, the model has long-run trade-offs so that international policy coordination is not merely a transient issue. The first-best optimum achieves the highest welfare a world planner can obtain and corresponds to no tax distortions, optimal quantities of money and unit marginal rates of substitution between public and private goods. However, the first-best optimum cannot be achieved in a decentralised market economy. Even though coordination in the absence of pre-commitment leads to no tax distortions and the optimal provision of public goods, lack of credibility leads to too high inflation rates and thus to too low holdings of real money balances. Coordination in the absence of pre-commitment is counterproductive, despite the fact that decentralisation leads to tax distortions. The reason is that decentralisation has a built-in disincentive to renege, for a unilateral surprise inflation tax induces a depreciation of the exchange rate and inflation costs, whereas international policy coordination has no such disincentive to renege. When the governments can make binding announcements about future monetary policy, welfare is improved so that the typical welfare ranking is first-best optimum, coordination with pre-

⁵When there is no discounting ($\delta = 0$) competitive decision-making is no longer time inconsistent, so that the pre-commitment and loss of leadership outcomes coincide and agents hold the OQM. This means that the public sector deficit and monetary growth are zero. International policy coordination under loss of leadership with $\delta = 0$ leads to larger stocks of real money balances, for governments are no longer impatient, and therefore leads to higher inflation. International policy coordination with pre-commitment and $\delta = 0$ increases real money balances to the OQM, yet revenues from seignorage and inflation are lower due to the fact that taxes are higher than with discounting. The result that coordination is worse than competitive decision-making under loss of leadership still holds when $\delta = 0$, since in the first case $U = -1.835$ and in the second case $U = -1.783$.

commitment, decentralisation with pre-commitment, decentralisation without pre-commitment and coordination without pre-commitment.

When the game between the governments and the private sectors is repeated indefinitely and the discount rate is not too high, the governments might be able to build up a reputation for sticking to announced policies [cf. Barro and Gordon (1983)] and therefore international policy coordination may be worthwhile. When one allows for 'severe' punishment strategies, it is possible, for a given discount rate and given punishment intervals, to sustain lower inflation rates than under the expectational rules considered by Barro and Gordon [Rogoff (1986)]. Similarly, reputations may develop and coordination may be productive when there is imperfect information and the game is repeated a finite number of times [cf. Backus and Driffill (1985)] or when the economies are stochastic [Currie and Levine (1986)]. Rogoff (1986) provides an excellent survey of reputational issues and points out that repeated-game models replace a 'cooperation' problem with a 'coordination problem'. Rogoff (1986) also considers severe punishment and trigger strategies, the problem of multiple equilibria, the implications of a continuum of decision-makers, the effects of private information, and extensions to multiple governments in an international context. Alternatively, in a richer model the problem of time inconsistency may disappear when the governments can issue a sufficiently rich maturity structure of government nominal and indexed debt [cf. Lucas and Stokey (1983), Persson, Persson and Svensson (1985)]. The interplay between time inconsistency, reputations and structure of financial assets on the one hand and the potential benefits from international policy coordination on the other hand, seem an interesting avenue for further research.

One could argue that, if policies can be pre-committed vis-à-vis the private sectors, then international policy coordination is probably also feasible. This argument might exclude the possibility of decentralisation with pre-commitment. If one also argues that coordination between two governments being feasible implies that pre-commitment is feasible, then this might rule out coordination without pre-commitment and therefore the paradox of the counter-productive nature of policy coordination discussed in Rogoff (1985) and in this paper might become a theoretical curiosum. However, it seems likely that it is easier to sustain cooperation between governments (via appropriate threat and punishment strategies or via a reputational mechanism) than to ensure a credible pre-commitment to announced government policies vis-à-vis a large group of atomistic private sector agents. Nevertheless, further research should attempt to explain what sustains international cooperation in the absence of pre-commitment.

In future research, the issue of credibility and subgame perfection should be further investigated. The loss of leadership solutions, employed in section 6, coincides with the open-loop Nash equilibrium solution, hence it is time

consistent but not necessarily subgame-perfect (or credible). The appendix shows how one can calculate the subgame-perfect solution, but in general this is a very difficult exercise as it is difficult to find the functional form of the value functions for nonlinear models and non-quadratic preferences. However, the problem may not be too serious for the particular model discussed in this paper. The credibility problem is due to the fact that there is an *ex post* incentive to levy a surprise inflation tax, which permits a cut in the distortionary income tax (and an increase in government spending). Since money is the only asset, any gain from a surprise inflation must be captured instantaneously and therefore it seems that there is no remaining incentive for a surprise inflation in the case of coordination with loss of leadership, especially as both taxes and government consumption are already at their first-best levels.⁶ For the case of competitive decision-making, it is not immediately clear what the effect of subgame perfection would be and this remains an important area for further research.

Another area of further research is to extend the model to three or more economies. Even if pre-commitment is feasible, international policy coordination between two countries may be counterproductive when there are third countries that tighten their monetary policies in response to attempts to coordinate policies in the two countries under consideration [cf. Canzoneri and Henderson (1986)]. Also, an investigation into the potential benefits of different regimes, e.g., fixed versus flexible rates, may be useful [cf. Canzoneri and Gray (1985), McKibbin and Sachs (1986)]. Finally, it is essential to make the model more realistic by allowing for bonds and capital accumulation [e.g. Lipton and Sachs (1983), Buiter (1986)] and perhaps for currency substitution. This may accentuate the results of this paper. An increase in the home monetary growth rate increases home inflation, reduces the world real interest rate and therefore increases output, capital and employment at home and abroad. Hence, each country wants to transfer the burden of lowering the world real interest rate to the other country, so that decentralised decision making with pre-commitment leads to too low monetary growth rates [van der Ploeg (1986a)].

Appendix: Credibility and subgame-perfect optimal taxation

It is not clear that the loss of leadership solution concepts employed in section 6 are credible. The reason is that the loss of leadership solution coincides with the open-loop Nash equilibrium solution, hence it is time consistent but not necessarily subgame perfect or credible. Indeed, Oudiz and Sachs (1985) argue that as time proceeds the expectations of the private sectors are not fulfilled as the private sectors' anticipations of future

⁶This argument is due to one of the referees.

government policy are not the loss of leadership outcomes. In other words, the governments have an incentive to announce policies that are different from the loss of leadership policies and that are believed by the private sectors. Such policies should be subgame perfect, so that announced policies should be rational to be carried out if called upon to do so at some later date. The resulting outcomes are based on feedback (or subgame-perfect) Stackelberg equilibrium solutions [Simaan and Cruz (1973b), Başar and Olsder (1982)] between each government and its private sector and can be derived with the aid of dynamic programming.

The representative household takes profits, the wage rate, the exchange rate and the inflation rate as given, so that its Hamilton-Jacobi-Bellman equation is given by

$$V_t^H - \delta V^H = \max_{c_D, c_M, l^*} [U(c_D, c_M, l^*, g, m) + V_m^H((1-\tau)(wl^* + z) - c_D - ec_M - \pi m_t^1)], \quad (42)$$

where $V^H(t, m; \tau, g, \pi, e)$ is the household's value function. It follows that $c_D = C_D(V_m^H)$, $c_M = C_M(eV_m^H)$ and, with LME, $l = L((1-\tau)V_m^H)$, so that a Markov-perfect equilibrium must satisfy:

$$\delta V^H + \beta_0 + \beta_1 \log(V_m^H) + \beta_2 \log(m) + \beta_3 m + \beta_4 V_m^H + \beta_5 m V_m^H = 0, \quad (43)$$

where $\beta_0 \equiv \alpha_1 \log(x_1) + \alpha_2 \log(x_2/e) + \alpha_3 \log(x_3/\beta(1-\tau)) + \alpha_4 \log(g) - (1-\alpha_4)$, $\beta_1 \equiv -(1-\alpha_4)$, $\beta_2 \equiv \alpha_5 m_F$, $\beta_3 \equiv -\alpha_5$, $\beta_4 \equiv -(1-\tau)\beta$ and $\beta_5 \equiv -\pi$. Eq. (43) is a non-linear ordinary differential equation, which can, in principle, be solved to give:

$$V_m^H = \phi(m, \tau, g, \pi, e). \quad (44)$$

The goods market equilibrium condition can be written as $V_m^H = A(\tau, g)$, which, together with the exchange market equilibrium condition, $e = E(V_m^H, V_m^{H*})$, yields $U(\cdot) = V(\tau, g, \tau^*, g^*) + v(m)$ [see eq. (19)]. Eq. (44) $V_m^H = A(\tau, g)$ can be solved with $e = E(\cdot)$ to give an expression for inflation,

$$\pi = \tilde{\pi}(m, \tau, g, \pi^*, g^*), \quad (45)$$

where $\tilde{\pi}_m = -\phi_m/\phi_\pi$, $\tilde{\pi}_i = -[\phi_i - ((e\phi_e/A) + 1)A_i]/\phi_\pi$, $i = \tau, g$, and $\tilde{\pi}_j = -(\phi_e/A)A_j^*/\phi_\pi$, $j = \tau^*, g^*$. It follows that the household or government budget constraint can be written as

$$\dot{m} = (1-\tau)\beta - \left(\frac{(1-\alpha_4)(\beta-g)}{\alpha_1 + \alpha_2 + \alpha_3(1-\tau)} \right) - \tilde{\pi}(\cdot)m \equiv \Omega(m, \tau, g, \tau^*, g^*), \quad (46)$$

When the two countries are engaged in competitive (Nash–Cournot) policy formulation, the Hamilton–Jacobi–Bellman equation of the home government is given by

$$V_t^G - \delta V^G = \max_{\tau, g} [V(\cdot) + v(m) + V_m^G \Omega(\cdot) + V_{m^*}^G \Omega^*(\cdot)], \quad (47)$$

where $V^G(t, m, m^*)$ is the value function of the home government. It follows that

$$V_t + V_m^G \Omega_t + V_{m^*}^G \Omega_t^* = V_g + V_m^G \Omega_g + V_{m^*}^G \Omega_g^* = 0 \quad (48)$$

must hold, which in symmetric Nash equilibrium yields $\tau = T_N(m, m^*, V_m^G, V_{m^*}^G)$ and $g = G_N(m, m^*, V_m^G, V_{m^*}^G)$. Substitution of $T_N(\cdot)$, $T_N^*(\cdot)$, $G_N(\cdot)$ and $G_N^*(\cdot)$ into (47) allows one, in principle, to solve for the asymptotic value function, $V^G(\tau, m, m^*)$ and consequently for the asymptotic feedback policy rules, $\tau = \bar{T}_N(m, m^*)$ and $g = \bar{G}_N(m, m^*)$. Upon substitution of these policy rules into (46), one can obtain the path of real money balances.

When the two countries engage in international policy coordination, the Hamilton–Jacobi–Bellman equation is given by

$$V_t^W - \delta V^W = \max_{\tau, g, \tau^*, g^*} [V(\cdot) + v(m) + V^*(\cdot) + v(m^*) + V_m^W \Omega(\cdot) + V_{m^*}^W \Omega^*(\cdot)], \quad (49)$$

where $V^W(t, m, m^*)$ is the value function of the world planner. It follows that

$$V_t + V_t^* + V_m^W \Omega_t + V_{m^*}^W \Omega_t^* = V_g + V_g^* + V_m^W \Omega_g + V_{m^*}^W \Omega_g^* = 0 \quad (50)$$

must hold, so that $\tau = T_C(m, m^*, V_m^W, V_{m^*}^W)$ and $g = G_C(m, m^*, V_m^W, V_{m^*}^W)$. Upon substitution of $T_C(\cdot)$ and $G_C(\cdot)$ into (50), one can, in principle, solve for the asymptotic world value function, $V^W(\tau, m, m^*)$, and the asymptotic policy feedback rules, $\bar{T}_C(m, m^*)$ and $\bar{G}_C(m, m^*)$.

Due to the analytical difficulties in solving for the value functions, it is very difficult to characterise the subgame-perfect outcomes and one has to resort to a complicated numerical algorithm. Such an algorithm effectively nests a Runge–Kutta routine for the numerical integration of non-linear ordinary differential equations within a Gauss–Newton routine for numerical optimisation under constraints, which in turn is nested within another Runge–Kutta routine. This presumably explains the popularity of the loss of leadership solution concepts, which are much easier to calculate. Cohen and Michel (1984), Miller and Salmon (1985) and Oudiz and Sachs (1985) suggest

an alternative algorithm for obtaining time-consistent strategies. It is based on iterations of the stable manifold associated with the perfect-foresight solution, (20). It is not clear whether their solution concept can be adapted to deal with non-linear models and non-quadratic preferences and whether it coincides with the subgame-perfect outcomes suggested above, but for the particular model discussed in this paper the problem may not be too serious. It should be pointed out that there are other reasonable outcomes that may be time consistent. One example is the open-loop Nash equilibrium or loss of leadership solution suggested by Buiter (1983), but this is not necessarily subgame perfect. Another example is based on reputational equilibria developed by Barro and Gordon (1983) and by Backus and Driffill (1985) [also see Rogoff (1986) for a survey]. Section 8 discusses these issues in more detail.

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